PROGRAM OF THE CONFERENCE ON ALGEBRAIC GROUPS, NANCY, FEBRUARY 2020

Raphael Achet: Unirational algebraic group.

Abstract: It is well known that if K is a perfect field, then any linear (smooth connected) algebraic K-group is unirational, and that any (smooth connected) unipotent K-group is rational. Over a nonperfect field k, a general linear k-group is not necessarily unirational. I'm going to talk about the geometry of the unirational k-groups with a focus on the unipotent case.

Let us begin with an example: if k'/k is a purely inseparable extension, we consider the quotient group $U = \mathcal{R}_{k'/k}(\mathbb{G}_{m,k'})/\mathbb{G}_{m,k}$ where $\mathcal{R}_{k'/k}(\mathbb{G}_{m,k'})$ denote the Weil restriction of the multiplicative group $\mathbb{G}_{m,k'}$. Then, U is a (smooth connected) unipotent group that is k-wound (i.e. U does not contains a subgroup isomorphic to the additive group $\mathbb{G}_{a,k}$) and unirational. This example have been studied by J. Oesterlé; the results he obtained suggest that these groups play a special role among the unirational unipotent k-groups (see *Nombres de Tamagawa et groupes unipotents en caractristique p* Chap. VI. Th. 3.1). Among other, I'm going to describe any commutative unirational unipotent k-group as the quotient of some unipotent k-group obtained by Weil restriction of the multiplicative group; this result confirms the intuition of J. Oesterlé.

Vladimiro Benedetti: The Cayley Grassmannian and its cohomology.

Abstract: The Cayley Grassmannian parametrizes four-dimensional subalgebras of the algebra of complexified octonions. It is a G_2-space (G_2 being the automorphism group of octonions) with three G_2 orbits. As a subvariety of a complex Grassmannian its description is quite explicit (it is a zero locus of a section of a vector bundle). Using this fact and equivariant techniques, Laurent Manivel studied this variety and gave a presentation of its cohomology. I will recall these results and then I will present a recent work in collaboration with L. Manivel: we compute the Gromov-Witten invariants of the Cayley Grassmannian and give a presentation of its small quantum cohomology. If time permits, I will discuss the validity of the Dubrovin conjecture for this variety.

Michel Brion: Automorphism groups of projective varieties.

Abstract: The automorphism group of a projective algebraic variety X is known to be a "locally algebraic group", generally with infinitely many components. The talk will discuss criteria for $Aut(X)$ to be linear algebraic; this holds for example if X has an action of a linear algebraic group with an open orbit, or an orbit of codimension 1.

Giovanna Carnovale: Jordan classes in Lie algebras and algebraic groups.

Abstract: Reductive algebraic groups and Lie algebras can be stratified by means of irreducible, locally closed, smooth unions of orbits of elements that have similar Jordan decomposition, called Jordan classes. They were introduced for the solution of representation theoretic problems: in the Lie algebra context they appeared in

the work of Borho and Kraft on sheets and the module structure of rings of regular functions on adjoint orbits. In the algebraic group context they made their first appearance in the work of Lusztig on the generalised Springer correspondence. After illustrating differences and similarities between the group and the Lie algebra situation, I will show how locally the two stratifications can be related and how to deduce geometric properties of the group stratification from properties of the Lie algebra one.

The talk is based on joint work with Filippo Ambrosio and Francesco Esposito.

Stéphanie Cupit-Foutou: Gromow-width of Bott-Samelson varieties.
Abstract:

Nguyen Duc Khanh: branching problem on winding subalgebras of affine Kac-Moody algebras $A_1^{(1)}$ and $A_2^{(2)}$.
Abstract: consider an affine Kac-Moody algebra $\mathfrak{g}$ with Cartan subalgebra $\mathfrak{h}$. Given Λ in the set P_+ of dominant integral weights of $\mathfrak{g}$, we denote by $L(\Lambda)$ the integrable highest weight $\mathfrak{g}$-module with highest weight Λ. For $\mu \in \mathfrak{h}^*$, we denote by $L(\Lambda)_\mu$ the corresponding weight space. Consider the support $\Gamma(\mathfrak{g}, \mathfrak{h})$ of the decompositions of the $L(\Lambda)$ as a $\mathfrak{h}$-module:

$$\Gamma(\mathfrak{g}, \mathfrak{h}) = \{(\Lambda, \mu) \,:\, L(\Lambda)_\mu \neq \{0\}\}.$$

Consider now the winding subalgebra $\mathfrak{g}[u]$ (for some positive integer u). The winding subalgebra $\mathfrak{g}[u]$ is isomorphic to lg but with a nontrivial embedding in $\mathfrak{g}$ depending on the parameter u. Given λ in the set $\dot{P}_+$ of dominant integral weights of $\mathfrak{g}[u]$, we denote by $\dot{L}(\lambda)$ the integrable highest weight $\mathfrak{g}[u]$-module with highest weight λ. Then the $\mathfrak{g}$-module $L(\Lambda)$ decomposes as a direct sums of simple $\mathfrak{g}[u]$-modules $\dot{L}(\lambda)$ with finite multiplicities. In this paper, we are interested in the supports of this decomposition, i.e., the set of pairs (Λ, λ) in $P_+ \times \dot{P}_+$ such that the integrable highest weight $\mathfrak{g}[u]$-modules $\dot{L}(\lambda)$ is a submodule of $L(\Lambda)$. We show that both $\Gamma(\mathfrak{g}, \mathfrak{h})$ and $\Gamma(\mathfrak{g}, \mathfrak{g}[u])$ are semigroups. Moreover, for the cases $A_1^{(1)}$ and $A_2^{(2)}$, we determine explicitly $\Gamma(\mathfrak{g}, \mathfrak{h})$. Finally, we describe explicit subsets of $P_+ \times \dot{P}_+$ where the two semigroups coincide.

Jacopo Gandini: Nipotent orbits of height 2 and involutions in the affine Weyl group.
Abstract: Let G be a semisimple algebraic group, with a fixed Borel subgroup B and maximal torus T in B. To any set of pairwise strongly orthogonal roots I will attach a nilpotent B-orbit in the Lie algebra of G, and will explain how the combinatorics of the involutions in the affine Weyl group of G relates to the geometry of such B-orbits. The talk is based on a joint work with P. Moseneder Frajria and P. Papi.

Auguste Hébert: Decomposition of principal series representations, for Kac-Moody groups over local fields
Abstract: Let G be a split reductive group over a local non-Archimedean field and H be its Iwahori-Hecke algebra. The sudy of the representations of H plays a key role in the representation therory of G. At the end of the 70's, Matsumoto introduced a class of representations of H, called "principal series representations".

Matsumoto and then S.Kato gave an irreducibility criterion for these representations. Let now G be a split Kac-Moody group over a non-Archimedean local field. Braverman, Kazhdan and Patnaik and then Bardy-Panse, Gaussent and Rousseau associated an Iwahori-Hecke algebra H to G in 2014. We can then define the principal series representations of H. I will talk about a partial generalization of Kato's criterion in this frameworks and of the decomposition of these representations, when they are reducible.

Dimitry Kfoury: Pieri Rule for the homology of the affine Grassmannian.
Abstract: The aim of this talk is to introduce the necessary elements of the proof of a Pieri formula for the homology of the affine Grassmannian in type C.

Bruno Laurent: Almost homogeneous varieties of Albanese codimension one.
Abstract: A variety is said to be almost homogeneous if it has a dense orbit under the action of an algebraic group. Almost homogeneous varieties are very symmetric objects, with a rich geometry, and have been much studied for the past fifty years; a famous class of them consists of toric varieties, when the acting group is a torus. In this talk, we are interested in varieties, defined over an arbitrary field, satisfying a geometric condition: having Albanese codimension 1.

A first part will be dedicated to some general results on the Albanese variety Alb(X) of a variety X. The Albanese codimension is defined to be dim X - dim Alb(X) and I will explain why this is an interesting geometric invariant when studying almost homogeneous varieties.

In a second part, I will present the classification of almost homogeneous varieties of Albanese codimension 1 and their equivariant compactifications.

If there is enough time, I will comment on the smoothness of the automorphism group of the equivariant compactifications.

Emmanuel Letellier: Fourier transforms on reductive groups.
Abstract: There is a natural Fourier transform on the space of complex valued functions on matrices $\mathfrak{gl}(n,q)$ over a finite field. This Fourier transform induces an operator on the space of functions of $GL(n,q)$ which satisfies interesting properties. For instance it distinguishes certain packets (Lusztig series) of irreducible representations of $GL(n,q)$. Given an arbitrary connected reductive group over a finite field, and a representation of its dual group $G^* \to GL(n)$, on can transfert the Fourier operator on functions of $GL(n,q)$ to a Fourier operator on functions on $G(q)$. In this talk we will discuss how to complete this Fourier operator into an involutive one. The motivation comes from the approach by Braverman-Kazhdan and Lafforgue to Langlands functoriality conjectures for local and global fields using Fourier transforms. This is a joint work with Grard Laumon.

Benoit Loisel: Construction of Λ-buildings associated to quasi-split reductive groups.
Abstract: In the 1970s, Kato (1978) and Parshin (1975) introduced higher-dimensional local fields that are a natural generalization of the usual local fields. Such a field $\mathbb{K}$ is equipped with a valuation $\omega : \mathbb{K}^* \to \mathbb{Z}^d =: \Lambda$. For such a field, Parshin constructed in 1994 a higher Bruhat-Tits building on which the group $\mathrm{PGL}_n(\mathbb{K})$ acts.

Independently, in 1994, Bennett introduced a notion of Λ-buildings for any totally ordered abelian group Λ that generalizes the notion of affine buildings previously introduced by Tits. Given a field $\mathbb{K}$ equipped with a valuation $\omega : \mathbb{K}^* \to \Lambda$, Bennett constructed such a Λ-building on which the group $\mathrm{SL}_n(\mathbb{K})$ acts.

Let Λ be a totally ordered abelian group. Let $\mathbb{K}$ be a Henselian field equipped with a valuation $\omega : \mathbb{K}^* \to \Lambda$. Let G be a quasi-split reductive $\mathbb{K}$-group. In 1972, Bruhat and Tits constructed a building on which the group $G(\mathbb{K})$ acts provided that Λ is a subgroup of $\mathbb{R}$. In a joint work with Hbert and Izquierdo, we deal with the general case where there are no assumptions on Λ and we construct a set on which $G(\mathbb{K})$ acts. It is a Λ-building, in the sense of Bennett.

In this talk, we firstly provide some examples of Λ-valued fields $\mathbb{K}$. Then we recall the definition of Λ-buildings given by Bennett and a list of some equivalent axioms stated by Bennett and Schwer and we explain how to attach a natural appartment $\mathbb{A}$ to a reductive $\mathbb{K}$-group G. We then explain that there are two different viewpoints on $\mathbb{A}$. We use both of them in order to construct a space $\mathcal{I}$ associated to the triple $(G, \mathbb{K}, \omega)$ and we prove that it is a Λ-building in the sense of Bennett.

Maxime Pelletier: On stable faces of the Kronecker cone.
Abstract: The Kronecker coefficients are an example of branching coefficients in representation theory, and can for instance be defined with representations of general linear groups over the field of complex numbers. One can also have a more geometric point of view on them, using spaces of sections of some line bundles on flag varieties.

We will be interested in the cone spanned by the non-zero Kronecker coefficients, and we will see that this geometric point of view allows to obtain results, using notably notions from Geometric Invariant Theory. We will principally focus on some particular faces of this cone, that we will call "stable". This name comes from a classical notion of stability of the Kronecker coefficients themselves."

Nicolas Perrin: VMRT of wonderful compactifications.
Abstract: (j.w. M. Brion and S. Kim) Varieties of Minimal Rational Tangents (VMRT) are important geometric invariants of varieties. In this talk I will explain how to describe the VMRT of wonderful compactifications of symmetric spaces and some connections to nilpotent orbits.

Kenny Phommady: About the polynomiality of semi-invariant algebras associated with a parabolic contraction in types A and C.
Abstract: Let G be a algebraic group and $\mathfrak{g}$ its Lie algebra. Let $Y(\mathfrak{g}) = \mathrm{S}(\mathfrak{g})^{\mathfrak{g}} \simeq K[\mathfrak{g}^*]^G$ be the algebra of elements of $\mathrm{S}(\mathfrak{g})$ for which the adjoint action of $\mathfrak{g}$ acts trivially. In invariant theory, one of long-standing questions is to know if $Y(\mathfrak{g}) = \mathrm{S}(\mathfrak{g})^{\mathfrak{g}}$ is polynomial or not. Positive answers (e.g. Chevalley in the reductive case) and negative ones (several counterexamples when $\mathfrak{g}$ is the centraliser of a nilpotent element in a reductive Lie algebra, for example Yakimova in 2007 in type E_8) have been given. Define then $\mathrm{Sy}(\mathfrak{g})$ as the algebra generated by elements of $\mathrm{S}(\mathfrak{g})$ for which the adjoint action of $\mathfrak{g}$ acts homothetically.

When $\mathfrak{q}$ is a parabolic contraction in type A or C, and in some cases in type B, Panyushev and Yakimova showed that the invariant algebra $\mathrm{Y}(\mathfrak{q})$ is polynomial. We present results about the polynomiality of $\mathrm{Sy}(\mathfrak{q})$ in types A and C, using Panyushev's and Yakimova's result.

Nicolas Ressayre: On the faces of the tensor cone of symmetrizable Kac-Moody Lie algebras
Abstract: It is a joint work with Shrawan Kumar. We are interested in the decomposition of the tensor product of two representations of a symmetrizable Kac-Moody Lie algebra $\mathfrak{g}$, and more precisely in the following tensor cone of $\mathfrak{g}$:

$$\Gamma(\mathfrak{g}) := (\lambda_1, \lambda_2, \mu) \in P^3_{+,\mathbb{Q}} : \exists N \geq 1 \text{ such that } L(N\mu) \subset L(N\lambda_1) \otimes L(N\lambda_2)\}$$

consisting in triples of dominant weights.

If $\mathfrak{g}$ is finite dimensional, $\Gamma(\mathfrak{g})$ is a polyhedral convex cone described by Belkale-Kumar by an explicit finite list of inequalities. This list of inequalities was proved to be irredundant: each inequality corresponds to a codimension one face. In general, $\Gamma(\mathfrak{g})$ is neither polyhedral, nor closed. Brown-Kumar obtained a list of inequalities that describe $\Gamma(\mathfrak{g})$ conjecturally. Here, we prove that each of Brown-Kumar's inequalities corresponds to a codimension one face of $\Gamma(\mathfrak{g})$.

Simon Riche: On tilting characters for reductive algebraic group.
Abstract: In joint work with Geordie Williamson we have proposed a few years ago a character formula for tilting modules in the principal block of a connected reductive algebraic group G over a field of positive characteristic p in terms of the p-canonical basis, under the assumption that p is at least the Coxeter number of G. This conjecture was later proved by the combination of several papers involving in particular Pramod Achar, Shotaro Makisumi and Geordie Williamson. In this talk I will present a proof of an extension of this formula to all blocks of G, which does not require any assumption on p. In principle, from this (using arguments of Andersen recently extended by Sobaje) one can deduce characters formulas for all simple G-modules. This is joint work with Geordie Williamson.

Fabio Tanturri: Orbital degeneracy loci and the geometry of the Coble cubic.
Abstract: In a recent series of a papers a new class of varieties, called orbital degeneracy loci, was introduced. They are modelled on any orbit closure in a representation of an algebraic group and generalize classical degeneracy loci of morphisms between vector bundles or zero loci of sections. In this talk I will introduce them and I will show how their language can be used to characterize the geometry of the Coble cubic hypersurface and the group law of an abelian surface. Based on joint works with V. Benedetti, S.A. Filippini, and L. Manivel.

Christian Urech: Actions of Cremona groups on CAT(0) cube complexes.
Abstract: To a variety X we can associate its group of birational transformations $\mathrm{Bir}(X)$. Recently, in geometric group theory, actions of groups on CAT(0) cube complexes have turned out to be a useful tool to study various groups. I will give a natural construction of CAT(0) cube complexes on which $\mathrm{Bir}(X)$ acts by isometries and explain how we can deduce new and old group theoretical and dynamical results from this action. This is joint work with Anne Lonjou.